An investigation of student understanding of the real image formed by a converging lens or concave mirror

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Student understanding of the real images produced by converging lenses and concave mirrors was investigated both before and after instruction in geometrical optics. The primary data were gathered through interviews in which undergraduates taking introductory physics were asked to perform a set of prescribed tasks based on a simple demonstration. The criterion used to assess understanding was the ability to apply appropriate concepts and principles, including ray diagrams, to predict and explain image formation by an actual lens or mirror. Performance on the tasks, especially by students who had not had college instruction in geometrical optics, suggested the presence of certain naive conceptions. Students who had just completed the study of geometrical optics in their physics courses were frequently unable to relate the concepts, principles, and ray-tracing techniques that had been taught in class to an actual physical system consisting of an object, a lens or a mirror, and a screen. Many students did not seem to understand the function of the lens, mirror, or screen, nor the uniqueness of the relationship among the components of the optical system. Difficulties in drawing and interpreting ray diagrams indicated inadequate understanding of the concept of a light ray and its graphical representation.

I. INTRODUCTION

This paper reports on an investigation of student understanding of the real image formed by a converging lens or concave mirror. This study, which extended over a period of two years (1982–1984), also included image formation by a plane mirror.[1] Conducted by the Physics Education Group at the University of Washington, this investigation was part of our ongoing effort to identify and address conceptual difficulties encountered by students taking introductory college physics.[2]

A number of investigators have sought to characterize how students who have had little or no formal instruction in optics think about light.[2–8] Several other studies with precollege students have concentrated on various aspects of geometrical optics.[9–13] However, the level at which college students understand image formation has not been systematically examined. In undertaking the present investigation, we were interested in determining whether students could respond only by rote or at a deeper level of comprehension to questions such as the following: What conditions are necessary for the formation of a real image? What is the function of the lens, the mirror, the screen? How does the relative position of these components affect the position of an image? What do we mean when we speak of light as coming to a "focus" at the image, and how does the use of the word in this context relate to the focal point of a lens or mirror? How can a ray diagram be used to predict the location of the image produced by an actual optical system?

All the questions above are tacitly addressed in the typical treatment of optics in introductory physics courses. In this paper, we present evidence that, although college students emerging from these courses may be able to give correct verbal responses to such questions, they are frequently unable to relate their knowledge to simple, but real, optical systems.

Most of the students who participated in our investigation were enrolled in the third quarter of algebra-based or calculus-based introductory physics at the University of Washington. The rest were in their second semester of algebra-based physics at West Virginia University. All the courses were taught by lecture. About half of the students had not yet studied geometrical optics in college. The other half had recently taken the course examination on that material. Of these, about half were enrolled in the optional accompanying laboratory course and had already completed the experiments in geometrical optics.

II. METHODS OF INVESTIGATION

It has been our experience that the most reliable indicator of conceptual understanding in physics is not what students say in response to a direct query, nor even how readily they can solve standard textbook problems, but rather how well they can apply their knowledge to a simple physical system that they can observe. Merely recalling statements from lecture or textbook may often suffice for an adequate response to a direct query about a concept. It is also possible for students who have memorized the requisite procedures, but not understood them, to use formulas and techniques, such as drawing ray diagrams, to solve the kinds of problems usually assigned in introductory physics courses.

Rather than trying to probe how students think about image formation by asking direct questions, we have approached this investigation from a laboratory perspective. The emphasis has been on the facility with which students can connect the concepts of geometrical optics with real world phenomena. To assess understanding, we have chosen as our primary operational criterion the ability of a student to apply the appropriate concepts and principles, including ray diagrams, to predict and explain image formation by actual mirrors and lenses. By examining how well students can cope with simple, but real, optical systems, we hope to gain some useful insights into how deeply they understand the ideas that underlie basic questions such as those listed near the beginning of this paper.

A. Primary data source

The data for this study were collected primarily through individual demonstration interviews, in which an investigator presents a series of tasks based on a demonstration that the student observes. In each task, the student is asked to make a prediction, or to perform an action, and then to give an explanation. In addition to asking a series of structured questions, the investigator may expand the questioning to clarify a student's response or to pursue an interesting point that may arise during the interview. The investigator also notes the student's actions and preserves all diagrams and calculations. Each interview usually lasts from 40 to 60 min, is recorded on audiotape, and is later transcribed for detailed analysis.

The tasks included in this investigation fall into three sets: plane mirror, converging lens, and concave mirror. Each student interviewed was presented with tasks involving the plane mirror,[1] followed by tasks with either the converging lens or concave mirror. In this paper, we discuss only the tasks on the converging lens and concave mirror. For all of these tasks, the demonstration consists of an object, a single lens or mirror, and a screen—all in full view of the student. The tasks that accompany each demonstration are in the form of questions about the nature, appearance, and location of the image under the observed conditions or with specified alterations to the apparatus.

Before presenting the tasks, the investigator asks the student to describe the demonstration. The student is then told to imagine a particular change in the optical system and to predict any changes that might occur as a consequence. The objective at this point is to elicit a relatively spontaneous prediction and verbal explanation. In their initial statements students often reveal ideas about the behavior of light and the formation of images ranging from undifferentiated preconceptions to fully developed concepts. Through further questioning, the investigator encourages the student to reflect on an initial prediction. If, in attempting to explain his reasoning, a student does not on his own initiative begin using ray diagrams, the investigator suggests the possibility. The determination of success or failure on a task is based on the ability of the student to apply the principles of geometrical optics, including the use of ray diagrams, to support a prediction that is correct or to revise one that is not.

No opportunity is provided during the interviews for the students to ascertain whether their responses on the tasks are correct. Any learning that might take place would interfere with the objective of examining how students draw on their prior knowledge or experience in attempting to perform the tasks. It is from such information that we hope to infer how students think about the real images formed by lenses and mirrors and to identify specific difficulties that they encounter. After the interview, we provide the opportunity for interested students to discuss the tasks with the investigator.

B. Supplementary data sources

Although the individual demonstration interview was the primary method of investigation in this study, we also obtained supplementary information from two other sources: group demonstration questionnaires and multiple-choice test questions. Each was administered in group settings to about 200 students. None of the students in the study participated in more than one of the three data-gathering procedures.

Experience acquired during the individual demonstration interviews guided the development of both types of group-administered tests and the evaluation of the student responses that they elicited. The group demonstration questionnaires consisted of modifications of some of the interview tasks. The demonstrations and tasks were presented to the group as a whole and the students responded in writing without individual interaction with the investigator. The questionnaires were administered before the relevant material in geometrical optics was presented in the lectures.

We were able to obtain additional data on the prevalence of certain difficulties by converting some of the interview tasks into multiple-choice questions and substituting sketches of the apparatus for the demonstrations. The questions were included as a small part of the final examinations in several sections of the large introductory courses. We considered the use of multiple-choice test questions in this investigation to be strictly exploratory.

C. Pre- and postinstruction groups

Approximately 80 students participated in the individual demonstration interviews as unpaid volunteers recruited from introductory physics courses at the University of Washington. Although a wide range of abilities was represented, most of the students had received grades above the class average in physics. Examination of the data revealed that the single factor that made the most difference in performance on the interview tasks was whether or not a student had completed the optics portion of the college physics course. Although there was not much difference in initial performance on the tasks, the students who had studied geometrical optics would often revise incorrect responses during the course of the interview. Consequently, they were eventually more successful on every task than the students who had not yet studied the material in their college course. We therefore decided to organize our analysis of the data in terms of a preinstruction group and a postinstruction group. For convenience, we will refer to the former as prestudents and the latter as poststudents.

We also considered several other factors that could contribute to differences in performance among the students, such as: completion of a high school physics course, enrollment in algebra-based versus calculus-based physics, enrollment concurrently in the optional accompanying laboratory course, and enrollment in sections taught by different instructors. It was apparent from the data that differences in performance that could be attributed to any one of the last four factors were much smaller than differences that could be attributed to pre-post status.

Better performance by the poststudents is, of course, a result that one might expect. What is surprising, however, is the degree of difficulty that these students had with the tasks and the number of errors they made that they did not recognize or could not correct.

III. CONVERGING LENS TASK

We began our investigation of student understanding of converging lenses by some exploratory questioning of individual students. We presented the following problem to ten poststudents: Given the focal length and the distance of an

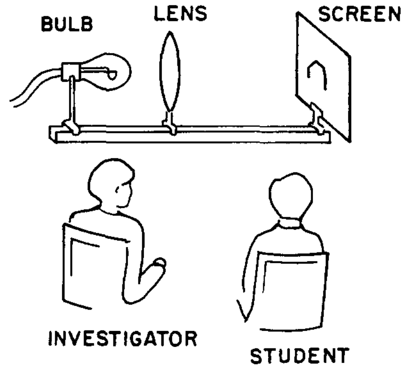

Fig. 1. Converging lens tasks 1–4. The investigator asks the student what would happen (1) if the lens were removed, (2) if part of the lens were covered, (3) if the screen were moved toward the lens, and (4) if the screen were removed.

object from a thin lens, find the image distance both by drawing a ray diagram and by using the lens equation. Specific numerical values were supplied. After the students had worked the problem, the investigator showed them an illuminated object mounted on an optical bench. A screen and a lens of the same focal length as that in the problem were on the table. The students were asked to arrange the lens and screen on the optical bench so that the distances between components corresponded to the numerical values that they had used or obtained in working the problem. We were interested in seeing whether students would have difficulty in relating the symbols used in the algebraic solution, the ray diagram, and the actual physical apparatus. All ten students readily accomplished the task and seemed to have little difficulty in going from the symbolic representations to the laboratory situation.

When, however, we began asking questions about the changes that would occur if the actual optical system were to be altered in certain specified ways, all the students demonstrated varying degrees of difficulty. To explore these systematically, we designed a set of four tasks based on a simple demonstration involving an object, a converging lens, and a screen. The demonstration used in all four lens tasks is shown in Fig. 1. The apparatus consists of an object, which in most of the interviews was an unfrosted light bulb with a horseshoe-shaped filament, a lens of diameter 7.5 cm with focal length 17 cm, and a translucent screen—all mounted on an optical bench. The bulb is 25 cm from the lens, which forms an enlarged, inverted image of the filament on the screen about 50 cm on the other side of the lens. The sharply defined image of the filament can be observed from either side of the translucent screen. In some of the earlier interviews, an illuminated slide of a cross-shaped figure was used as the object.[14] Both the student and the investigator are seated about 50 cm transverse to the lens axis and view the side of the screen facing the lens. The tasks were administered in the sequence in which they are discussed below, although not every student did every task. The results of all four are summarized in Table I, in which the percentages have been rounded to the nearest 5%.

A. Converging lens task 1

After the student has described the image on the screen, the investigator presents the first task by saying: "Suppose I were to remove the lens, leaving the object and screen where they are. Would anything change?"

The immediate response of the majority of the students interviewed was that the image on the screen would become erect. From the explanations offered, it seemed that only a few students recognized that without the lens there would be no image. During further questioning, most of the students continued to maintain that there would still be an image, although it might be fuzzy. Some stated that without the lens the light would just travel outward from the bulb in straight parallel lines and form an erect image on the screen. Asked in this context to explain the purpose of the lens, many students said it was to invert and perhaps change the size of the image. The poststudents did only marginally better than the prestudents in predicting what would happen if the lens were removed. The following interview excerpt illustrates a typical incorrect response.
(S: Student; I: Investigator)

S: ...the horseshoe would return back to its original—to the shape that you would expect it to be without the lens, which is right side up.

I: Can you explain your reasoning?

S: The lens has the effect of bending the light rays, or actually it just moves them towards each other, so that they cross at a point, which ends up flipping the image over and turning it backwards.

I: And what happens without the lens?

S: Without the lens the rays just follow a normal straight path.

This prediction, of course, was completely contrary to the daily experience of the students. It seemed to us that they all must have observed that light bulbs do not generally form images on surfaces, such as walls or ceilings. We decided to investigate the possibility that the incorrect responses might be due to some idiosyncrasy in the apparatus, rather than to a lack of understanding of the function of a lens. Consequently we administered the task as a group demonstration questionnaire at West Virginia University to two different groups of students. The demonstrations that accompanied the task differed slightly in the two instances. For one group, the same unfrosted bulb, with its clearly defined filament, served as the object. For the other, a standard frosted incandescent bulb was used. With the lens in place, the inverted image of the frosted bulb was clear and bright.

The predictions of both groups of students are shown in Table II, in which the percentages have been rounded to the nearest 5%. Each group contains both pre- and poststudents. Whereas only about 20% of the students predicted that the image of the unfrosted bulb would disappear, 60% of the students gave the right response for the frosted bulb. Therefore, use of the unfrosted bulb appears to have contributed to the difficulty the students had on this particular task. However, the fact remains that even when the difference in bulbs is taken into account, there were still a significant number of students who did not recognize that the presence of the lens was necessary for the formation of an image.

Although it is difficult to account with certainty for the difference in results obtained with the frosted and unfrosted bulbs, the comments of the students on this and other tasks suggest a possible explanation. Perhaps the well-defined shape of the filament in the clear bulb might have encouraged students to think of light as leaving the filament in straight parallel lines and forming an image upon

Table I. Student responses on the first four converging lens tasks. Percentages have been rounded to the nearest 5%. Correct responses are indicated by an asterisk (*).

Task 1

Question: If lens were removed, would anything change on the screen?

Response	Preinstruction: Individual demonstration interviews (N = 38) (%)	Preinstruction: Group demonstration questionnaire (N = 172) (%)	Postinstruction: Individual demonstration interviews (N = 22) (%)	Postinstruction: Multiple choice test question (N = 222) (%)
No image*	35	35	50	55
Erect image	60	60	40	45
Other	5	5	10	0

Task 2

Question: If top half of lens were covered, would anything change on the screen?

Response	Preinstruction: Individual demonstration interviews (N = 36) (%)	Preinstruction: Group demonstration questionnaire (N = 172) (%)	Postinstruction: Individual demonstration interviews (N = 23) (%)	Postinstruction: Multiple choice test question (N = 222) (%)
Entire image remains*	0	10	35	25
Half of image vanishes	95	90	55	75
Other	5	0	10	0

Task 3

Question: If the screen were moved toward the lens, would anything change on the screen?

Response	Preinstruction: Individual demonstration interviews (N = 37) (%)	Preinstruction: Group demonstration questionnaire (N = 172) (%)	Postinstruction: Individual demonstration interviews (N = 19) (%)	Postinstruction: Multiple choice test question (N = 170) (%)
No image*	10	5	40	35
Image changes size and becomes somewhat fuzzy	20	10	35	↑ 65[a] ↓
Image changes size and remains clear	70	85	25	

Task 4

Question (while student actually looks at image): Where is the image located?

Response	Preinstruction Individual demonstration interviews (N = 15) (%)	Postinstruction Individual demonstration interviews (N = 21) (%)
At same position as screen*	0	25
Somewhere between lens and eye (nonspecific)	10	25
In (or on) lens	90	20
Between light bulb and lens	0	30

[a] It was not possible to distinguish between these two categories of response.

Table II. Student responses on group demonstration questionnaire for Converging Lens Task 1 with both frosted and unfrosted bulbs used as the object. Pre- and postinstruction results have been combined. Percentages have been rounded to the nearest 5%. Correct responses are indicated by an asterisk (*).

Task 1 Question: If lens were removed, would anything change on the screen? Student's prediction	Frosted bulb ($N = 53$) (%)	Unfrosted bulb ($N = 45$) (%)
No image*	60	20
Erect image	25	60
Other	15	20

being intercepted by the screen. (The results obtained with the slide of the cross-shaped figure, also a well-defined object, support this interpretation.) Under these circumstances, the students might have been more inclined to think of the lens as essential only to invert and change the size of the image. On the other hand, with the more extended light emitting surface of the frosted bulb, the students might be more likely to think of light as emanating from the bulb in all directions. A lens might then seem more necessary for focusing the light to form an image.

We decided to examine the possibility that the use of the frosted bulb would also make a difference on the other lens tasks that are described below. Accordingly, we substituted a frosted bulb in the demonstration and administered those tasks on a group questionnaire to a subset of the same students at West Virginia University. The type of bulb seemed to make no significant difference.

B. Converging lens task 2

The second lens task explores the ideas students have about the contributions of various parts of the lens surface to the formation of an image. The investigator holds a piece of opaque cardboard above the lens but does not cover any part of it. He then poses the following question: "Suppose I were to bring this cardboard down and cover the upper half of the lens, leaving the lower half uncovered. Would anything change on the screen?"

In this case the entire image remains intact and only its brightness decreases. Since light leaves in all directions from every point on the object, light from every point on the object passes through every part of the lens. Therefore, any portion of the lens would be sufficient to form a complete image. Covering part of the lens only diminishes the amount of light from the object that is brought to a focus by the lens.

A majority of the students, however, predicted that half the image would disappear. Many of the prestudents claimed that the half that would vanish would depend on whether the front upper half or the back upper half of the lens were covered. Their comments seemed to indicate that they were thinking of the image as actually entering the lens and being inverted inside. This idea is expressed in the following excerpt from an interview with a prestudent. In this interview, an illuminated slide of a cross-shaped figure was used as the object instead of the bulb filament.

S: ...the image of the cross comes down into the lens as we see it, is turned upside down, and then comes out. So if you put the cover between the light source and the lens, it didn't have time to turn upside down yet, so what appears as the top of the light source is the bottom of the image that you see, so therefore you won't see the bottom. Likewise, if you put the cover on the top half of the lens again, only this time in between the image and the lens, what you're essentially covering is the bottom half of the cross, because it's been turned upside down.

Most of the poststudents who answered incorrectly appeared to be convinced that something would need to change if half the lens were covered. The most common immediate response seemed to be based on the belief that, if half the light were blocked from passing through the lens, half the image would vanish.

The ray diagrams produced by many of the students often reinforced their intuition that half the image would disappear. In the diagram in Fig. 2, the student locates the image by drawing the two special rays from the top of the object: one, parallel to the principal axis, is refracted through the focal point; the other through the center of the lens, passes through undeviated. From this essentially correct diagram, the student concludes that if the upper half of the lens were to be covered, the two special rays would be blocked. Consequently the bottom half of the image would be missing. Students who drew similar diagrams seemed to believe that these two special rays are necessary to form an image. In fact, these two rays are sufficient but not necessary for locating the position of the image. Perhaps some of these students might have replied that the image remains intact if they had also considered the third of the special rays that can be used to locate its position. That ray leaves the top of the object, passes through the focal point, enters the bottom portion of the lens, and emerges parallel to the principal axis.

Probably contributing to the difficulty with the second lens task was a failure to distinguish between locating and forming an image. This distinction is perhaps made more difficult by the way we speak of light rays in everyday life as, for example, when we refer to the Sun's rays. The students often did not seem to be aware of the difference between the concept of a light ray and a narrow beam of light. A ray is an abstraction, part of a model to be used according to certain rules to locate an image point, rather than an actual physical entity.

Some of the incorrect ray diagrams the students drew in responding to the second lens task consisted of two parallel rays leaving the object, one from the top and one from the bottom. These students predicted that since the top ray would be blocked by the cardboard, it would not reach the screen. Therefore, the lower half of the image would van-

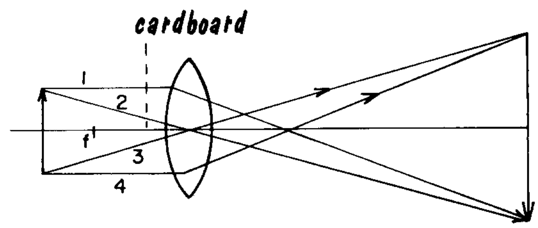

Fig. 2. Ray diagram drawn by student to justify prediction that half the image would disappear if half the lens were covered.

ish. The diagrams these students drew did not include any rays not parallel to the principal axis. This omission suggests that instead of conceiving of light as leaving in all directions from each point on the object, the students might be thinking that parallel rays emanate from an object, maintaining its shape as they travel through space to form an image. To explore this possibility, we designed two additional questions that we included in the last third of the interviews.

Each of the questions involved showing the students an opaque piece of cardboard in which a circular hole had been cut. The students were asked if anything would change on the screen if first one cardboard mask, and then the other, were placed in front of the lens so that the center of the hole in each mask was aligned with the center of the lens.

The first cardboard mask shown to the students had a hole that was a little larger than the horseshoe-shaped filament in the unfrosted light bulb. All but one out of twelve students predicted that there would be no change in the image, i.e., that the image would appear in its entirety. No one mentioned that the brightness would decrease. A number of the students specifically commented that they were basing their prediction on the relative size of the filament and hole.

We next showed the students the other piece of cardboard, in which the circular hole was considerably smaller than the size of the filament. About half of the students responded that the image would vanish. It was clear from their behavior that they were making a visual comparison between the size of the filament and the diameter of the hole. Almost all of the students who used parallel rays exclusively in their ray diagrams reasoned that rays from the filament would now be totally blocked by the cardboard.

C. Converging lens task 3

For the third lens task, the investigator asks the student the following question: "Suppose I were to move the screen toward the lens. Would anything change on the screen?" An acceptable response would be that, as the screen position begins to change, the image immediately becomes blurred and quickly disappears altogether. Except in the vicinity of the image position, all that can be seen on the screen is diffuse light of varying shape and area. In many of the interviews, especially among the prestudents, the prediction was made that, as the screen was moved toward the lens, the image would remain clear with only its size changing. Other students stated that the image might become "fuzzy" if the screen were moved. Their remarks indicated that they seemed to think that the image would be "out of focus" rather than disappear.

Many of the remarks made by the students indicated that the function of the screen was widely misunderstood. Often they did not think of it as a diffuse reflector or transmitter that, when located at a particular position for a given object distance, makes it possible for an observer not looking along the axis of the lens to see the image. Instead, they seemed to believe that an image can be seen on a screen no matter where it is placed along the axis. In some cases, this claim seemed to be buttressed by a misinterpretation of the experience of watching someone else use a slide projector. The students may have remembered that, in order to make the image larger, the screen had to be moved further from the projector. However, they did not also recall that it was necessary simultaneously to refocus the projector by changing the object distance.

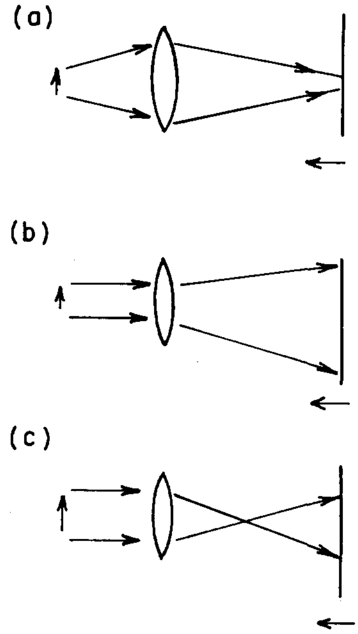

Fig. 3. Ray diagrams drawn by students to justify their predictions of what happens to the image if the screen is moved toward the lens. Student predictions: (a) "the image gets bigger," (b) "the image gets smaller," (c) "the image gets smaller, goes to a point, and then inverts."

In justifying their responses on the third lens task, some of the students drew sketches like those shown in Fig. 3. In all three cases, two parallel, or nearly parallel, rays leave the object, one from the top and one from the bottom. As they pass through the lens, the two rays are bent either toward or away from the principal axis. Students who thought that the image would get bigger as the screen was moved toward the lens generally drew diagrams similar to that in Fig. 3(a). By far, the most common incorrect response was that the image would become smaller. Usually the sketch that accompanied this prediction resembled the drawing in Fig. 3(b). Some students, however, drew a ray diagram similar to the drawing in Fig. 3(c) and predicted that as the screen was moved toward the lens the image would get smaller, go to a point, invert, and then become enlarged. They indicated that a clear image would appear on the screen if it were placed at any position. Many seemed to think that the size of the image would be delimited and its orientation determined by the two light rays that they had drawn from the top and bottom of the object.

It should be noted that the drawing in Fig. 3(c) resembles the ray diagram for an object at infinity and shows the procedure for locating the focal point of the lens. The drawing of such a diagram for an object that is relatively close to the lens has several possible implications. The student may not distinguish between the focal point of the lens (at which rays parallel to the principal axis from an infinitely distant object are brought to a focus) and a point on the image (at which rays from a point on a nearby object are brought to a focus). The student may think that all rays from the object travel parallel to the principal axis and cross it at the same point. It also seems clear from the drawing that the student does not realize that it is necessary to draw two rays from each object point to locate the corresponding image point. The failure to recognize that there is a unique position for the image probably contributes to the belief that the image can be seen on a screen regardless of its position.

D. Converging lens task 4

The fourth lens task was the last to be presented in the interviews in which the converging lens was used. While the student is looking at the image on the screen, the investigator asks the student if he would still be able to see the image from his present position if the screen were removed. Virtually all the students stated that the screen would be necessary for reflecting the image so that it could be seen. Several of the prestudents commented, however, that if the screen were removed they might be able to see the image on the wall, which was several meters beyond the screen.

The students were then asked whether they would be able to see an image if the screen were removed and they were free to stand up and move around to another position. Many of the students, especially the prestudents, seemed to have difficulty in understanding the question. Their remarks indicated that they could not conceive of an image as existing in space, independent of a surface. The majority of the students, both pre and post, said either that they would not be able to see an image or that they might be able to see an image if they could place their eye at the screen position. Several of the students who gave the latter response seemed to think of their eye as simply replacing the screen.

At this point in the interview, the investigator actually removed the screen and directed the student to move about two meters beyond the initial screen position and to look along the lens axis toward the lens. With this guidance, almost all of the students were able to see the aerial image. Many appeared surprised that they were able to see anything.

When the investigator asked for the location of the image, only a very few students were able to state correctly that the image was located at the same position that the screen had been. The rest of the students gave a variety of answers. The prestudents, especially, tended to say that they thought the image was at or in the lens. Their judgment may have been influenced by the belief discussed above that an image has to be on some surface.

The following excerpt from an interview with a poststudent is illustrative.

> I: As you look at that image, where is it located?
> S: (Long pause) It looks like it's in front of the lens when I look at it, right at the lens itself.
> I: Some people have said the image is right here where the screen was before. Is that where you think it is?
> S: No. It's not at the screen because...there's nothing for the light rays to focus on. There'd be no point for them to focus on, no surface at all without the screen there.

Seeing the image against the lens as background, coupled with a belief that an image can only be seen on a surface, may have suggested to some of the students that the lens was "framing" the image. These students seemed unaware that parallax between the lens and the image could have been used as a test to eliminate the lens surface as a possibility for the location of the image.

To help overcome the tendency to interpret the lens as framing the image, the investigator moved the screen in and out of the light beam several times during some of the interviews. We expected that by alternately seeing the aerial image and the screen image, the students would notice that the position was the same. However, very few who did not already know the correct image position changed their answer in response to this strong hint. Below is a typical excerpt from another interview with a poststudent who had observed the image while the screen was alternately moved in and out of the beam, but who still was unable to locate the image properly.

> I: ...Are you saying that when I put the screen here, the image is formed on the screen?
> S: Yes, there is an image there.
> I: But without the screen, is there still an image there?
> S: I would say that there's not an image there, because the light rays coming from the lens and intersecting are not intersecting to form an image. They intersect, but they don't form an image unless they're stopped by the screen and reflected back...

Fewer than one-fourth of all the pre- and poststudents were eventually able to state correctly that the aerial image was located at the same position as the screen had been. In trying to justify a correct response, most could offer no explanation other than a statement that "it looked that way."

Group demonstration questionnaire

The group demonstration questionnaire was administered to a large section of the calculus-based introductory physics course before the lectures on geometrical optics began. To the extent that it was feasible, the investigator presented the demonstrations and tasks to the class in the same way as in an interview. Since it was not possible to respond to individual requests for clarification, the students were also provided with written copies of the questions. As in the interviews, the drawing of ray diagrams was actively encouraged. Enough space was provided on the questionnaire for the students to write as much as they wished.

Table I presents the results from the questionnaire for the first three converging lens tasks. The fourth is not included because it could not be presented in the same way on the questionnaire as it was during the interviews. In the latter, the students were actually looking at the aerial image when they were asked to find its location.

It was apparent from an examination of the written responses, including the ray diagrams drawn by the students, that the results were similar to those obtained during the individual interviews. All the same difficulties appeared, and no new ones surfaced that had not been previously detected. Thus we were able to draw on results from the interviews to establish categories of student response and to develop a coding scheme that we could use to classify the results from the questionnaires. As can be seen from the prestudent data in Table I, the patterns of response on the questionnaires for the first three lens tasks were not significantly different from those obtained in the individual interviews. Furthermore, a more detailed analysis of the responses indicated that essentially the same information about conceptual understanding was obtained from group administration of the tasks as was obtained in individual interviews.

Multiple-choice test questions

Table I also displays the results obtained from administering multiple-choice test versions of the converging lens tasks as part of the final examinations in algebra-based and calculus-based introductory physics. Only the first three tasks are included since the fourth converging lens task could not be presented in the same way on a multiple-choice test as in the interviews.

For the third task, the correct response was not explicitly included as one of the choices but subsumed under "none of the above." Since all the other choices involved an image remaining on the screen in some form, it seems reasonable to assume that the students who selected this response did so because they believed that the image would vanish if the screen were moved toward the lens. Therefore, in Table I, we have placed the "none of the above" responses in the category labeled "no image." We have also expressed as a single percentage all incorrect responses to this question because, unlike the situation with the interviews, it was impossible to distinguish between students who thought the image would remain and be clear and those who thought it would remain but be "out-of-focus."

The results from the multiple-choice test questions are more difficult to interpret than the results from either the individual demonstration interviews or the group demonstration questionnaires. It is clearly not possible to ascertain how the students might have responded without the choices present. Nor can any conceptual details be extracted from the responses. Nevertheless, in spite of these shortcomings, the test results did provide us with some useful information. The multiple-choice versions of the converging lens tasks seem to have produced roughly the same profile of student response as was obtained on the individual demonstration interviews and on the group demonstration questionnaires.

The large group tests lend support to the generalizability of the findings from the interviews. The same qualification, however, applies in the case of both the multiple-choice questions and the group demonstration questionnaires. Both were designed on the basis of the results obtained from a large number of individual interviews.

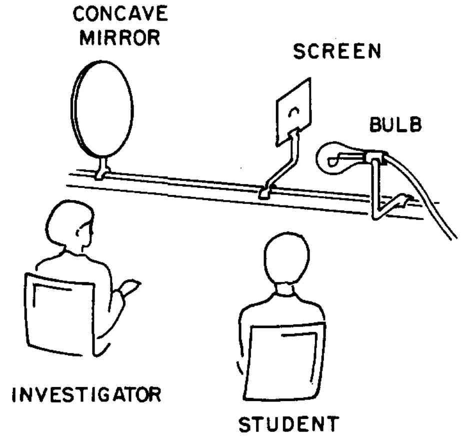

Fig. 4. Concave mirror tasks 1–4. The investigator asks the student what would happen (1) if the concave mirror were replaced by a plane mirror, (2) if part of the mirror were covered, (3) if the screen were moved toward the mirror, and (4) if the screen were removed.

Table III. Postinstruction student responses to corresponding questions on individual demonstration interviews with concave mirror and converging lens. Percentages have been rounded to nearest 5%. Correct responses are indicated by an asterisk (*).

Tasks on concave mirror/ converging lens	Responses	Concave mirror (%)	Converging lens (%)
Task 1:		($N = 12$)	($N = 22$)
Replace concave mirror with plane mirror/ remove lens. Would anything change on screen?	No image*	65	50
	Erect image	35	40
	Other	0	10
Task 2:		($N = 13$)	($N = 21$)
Cover half of mirror/ lens. Would anything change on screen?	Entire image remains*	45	20
	Half of image vanishes	30	70
	Other, or unsure	25	10
Task 3:		($N = 13$)	($N = 19$)
Move screen toward mirror/lens. Would anything change on screen?	No image*	40	40
	Image changes size and becomes "fuzzy"	45	35
	Image changes size and remains clear	15	25
Task 4:		($N = 12$)	($N = 23$)
Remove screen and look at image. Where is image located?	Same position as screen*	60	25
	On mirror (lens)	15	15
	Other location	25	60

IV. CONCAVE MIRROR TASKS

To determine whether the difficulties with the four converging lens tasks were primarily due to some idiosyncrasy in the apparatus or to a more basic lack of understanding of the nature of a real image, we designed four tasks involving a concave mirror that were intended to be analogous to the converging lens tasks. The concave mirror tasks were administered only to poststudents. It became apparent after a small number of interviews that the use of a concave mirror, instead of a converging lens, did not affect the general nature of the results. The kinds of difficulties exhibited by students on both types of tasks proved to be similar. We therefore decided that a small number of interviews on the concave mirror tasks would be sufficient. Although they yielded little new information, we found that some of the interviews on the concave mirror tasks served to deepen our insights into certain difficulties that the students encountered with the corresponding lens tasks.

The demonstration for the concave mirror tasks consists of a luminous object, a concave mirror and a screen—all mounted on an optical bench, as shown in Fig. 4. The object is the same as that used in the converging lens tasks, an unfrosted light bulb with a horseshoe-shaped filament. The diameter of the mirror is 15 cm and the focal length is 30 cm. The bulb is located about 80 cm from the mirror. A diminished, inverted image of the filament can be seen clearly from either side of the translucent screen, which is located between the mirror and bulb about 50 cm from the mirror. The bulb and the screen are slightly displaced from the axis of the mirror so that the screen does not prevent light from the bulb from reaching the mirror. Both the student and the investigator are seated about 50 cm transverse to the axis of the mirror, with the investigator to the left of the screen and the student to the right. They view the image from opposite sides. Three of the four concave mirror tasks are exactly analogous to three of the converging lens tasks; the remaining task differs somewhat from the corresponding lens task but has elements that are similar. The results of all four concave mirror tasks, along with the corresponding lens tasks, are summarized in Table III, in which the percentages have been rounded to the nearest 5%.

As can be seen from Table III, student performance on the concave mirror tasks was somewhat better than on the corresponding lens tasks. The differences in apparent difficulty of the two sets of tasks may have been partly due to certain differences in the demonstrations, e.g.: the diameter of the mirror was twice that of the lens; the mirror produced a much smaller image than the lens; an observer looking along the the principal axis of the mirror could see the actual bulb while viewing its image, but an observer could not see both the bulb and its image while looking along the principal axis of the lens. A few other factors not related to the apparatus might also have had some effect.

The students had much the same difficulties in interpreting a real image whether it was produced by a converging lens or a concave mirror. Because of the similarity between both the questions asked and responses obtained, we do not discuss the tasks with the concave mirror in as much detail as those with the converging lens. When, however, there are significant differences between a mirror task and a corresponding lens task, we comment specifically.

A. Concave mirror task 1

The investigator begins this set of tasks by first asking the student to describe the image on the screen. After the student has done so, the investigator presents the three concave mirror tasks that are exactly analogous to the last three converging lens tasks. When these are completed, the student is asked what would happen to the image if the concave mirror were removed and replaced by a plane mirror. Since this task is similar, although not quite analogous, to the first converging lens task, we refer to it as the first concave mirror task. However, it was actually presented last.

Four out of the twelve students interviewed predicted that, if the concave mirror were replaced with a plane mirror, there would be an erect image on the screen. Their explanations implied that they thought that the primary function of the concave mirror was to invert the image. They did not realize that, if a plane mirror were substituted, no image would be formed on the screen. Instead, they commented that the light from the bulb would travel in straight parallel lines to the mirror, be reflected in the same way, and on reaching the screen form an erect image. This type of reasoning is reminiscent of that used by other students on the corresponding converging lens task. On both tasks, the students seemed to be ignoring the role of the converging lens or concave mirror in focusing light from every point on the object to a corresponding point on the image.

B. Concave mirror task 2

In the second concave mirror task, the investigator holds an opaque piece of cardboard above the mirror. The student is asked to predict any changes that would occur on the screen if the cardboard were to be placed in front of the upper half of the mirror. Almost one-half of the students responded correctly that the image would remain intact. This particular task was one of the two in which a much greater percentage of students were successful with the concave mirror than with the converging lens.

Some of the students who eventually responded correctly initially said that half the image would vanish but changed their minds after finding that they could not draw a ray diagram that would support this prediction. This situation differs from that for the corresponding converging lens task, in which a correct ray diagram often reinforced the belief that half the image would vanish. In that case, for an object above the principal axis, the ray drawn through the center of the lens might appear to be blocked by the cardboard. However, in the arrangement used for the concave mirror, the object is located beyond the center of curvature of the mirror. Thus, in the ray diagram for an object above the principal axis, the ray drawn through the center of curvature could not possibly be blocked by the cardboard if it were to be placed so that it covered the top half of the lens.

Two additional questions were included on the second concave mirror task. The students were shown two masks, each with a circular aperture. One of the holes was larger than the filament and one smaller. The students were asked to imagine that the masks were placed in front of the mirror, one at a time, so that the center of the aperture was coaxial with that of the mirror. The results were very similar to those obtained on the corresponding converging lens task for the two masks of different diameters. For the mask with the larger hole, virtually all the students predicted that there would be no change in the image formed by the concave mirror. For the hole that was smaller than the filament, about half of the students predicted that the image would vanish. The following interview excerpt illustrates the type of response given by students who did not recognize that any part of the reflecting surface would be sufficient to form an image. The excerpt begins just after the investigator has shown the student the mask with the larger hole.

> S: It depends...we may get the whole image, and we may not, depending on where these beams of light were striking ...light is going off in all directions, but the ones needed to make the image are essentially parallel; and this is about so big...it's bigger than the filament. I say you'd get the same image.
>
> I:...so, nothing would change on the screen?
>
> S: Yeah, I would say, nothing would happen, because we only need the rays that are going to make that image, to see the image.
>
> I: What if I use this mask instead? It has just a little tiny hole in the center. If I put that in front of the mirror, would anything change?
>
> S:I don't think you'd get anything back, except, like, maybe just a little bit of fuzzy light.
>
> I: I can make this filament brighter. Watch the filament. If I were to make it much brighter, do you think it would make a difference?
>
> S: No, I still don't think it'd make much difference.
>
> I: And the reasoning?
>
> S: ...Because the hole is too small...The part of the mirror that's exposed has to be at least as big as the filament to get an image back, or get the whole image back.

Instead of comparing the relative sizes of the filament and small hole, the better students generally tried to apply a ray-tracing procedure. Their efforts in this analysis were often stymied, however, by the belief that if the mask should block the special rays used to locate the image, the image could not be formed. Following is an excerpt from an interview with a student who initially demonstrates this concern but who eventually predicts correctly that the image would remain intact. The statement quoted was made shortly after the student had reasoned successfully that the image would remain if the mask with the small hole were placed in front of the concave mirror.

> S: ...I think I have a problem because like when you draw these diagrams, you tend to always draw the same kind of rays. You draw one parallel and it reflects back through the focal point and you draw one through the focal point and it reflects back parallel. But you tend to forget that there are lots of other rays. I mean all those rays are hitting the mirror and intersecting. I guess that's why I initially said that if that hole was too small that there would be no image, because I was thinking of these typical rays that you always draw...like this one coming in parallel. The hole is smaller than that. It wouldn't ever hit the mirror, but there are still lots of other rays hitting.

The excerpt above illustrates the important contribution that bright students can make to an investigation of conceptual understanding. At first, such students often seem to be as perplexed by a particular task as less able students. We found, however, that as they struggled toward a solution, the better students could frequently articulate clearly the nature of some of the difficulties that all of the students were encountering.

C. Concave mirror task 3

In the third concave mirror task, the student is asked to describe any changes that might take place if the screen were moved toward the mirror. Fewer than half of the 13 students to whom this mirror task was presented realized that the image would vanish. Student performance on this task was similar to that of the poststudents on the third converging lens task. On both versions of the tasks, students who maintained that the image would increase or decrease in size and become somewhat "fuzzy," generally could give no coherent explanation for their prediction and seemed unsure of their answer.

D. Concave mirror task 4

For the fourth concave mirror task, the students were asked if they would still be able to see the image from their seated position if the screen were removed. All predicted that they would no longer be able to see the image. When asked if they would be able to see the image if they were free to stand up and move around to another position, most of the students said that they would not. The reasoning they gave seemed to indicate that they thought that a surface was necessary for seeing an image. When, at the investigator's suggestion, these students actually looked along the axis of the mirror after the screen was removed and saw the image, many seemed surprised. The following excerpt from a transcript shows a typical student reaction on seeing the image.

> S: ...it looks to me like where the screen used to be. It just does!
>
> I: ...You're surprised?
>
> S: Well, yeah, a little bit (laughs).
>
> I: What is surprising you?
>
> S: I guess it surprises me that I can see light (image) where there's nothing for it to be reflected off of. ...there's no surface there.

More than half of these poststudents were able to locate the aerial image at the same position as the screen had been. As Table III indicates, the discrepancy in performance between the converging lens and concave mirror versions of this task was greater than for any of the other tasks. A number of factors may have combined to make the concave mirror version of this task simpler than the corresponding converging lens task. The larger size of the mirror and the greater difference in relative size between the image and mirror, compared with the image and lens, both served to decrease the visually perceived "framing" effect that might have influenced students in the case of the converging lens. It should also be noted that the bulb could be observed only through the lens in the converging lens task. In the concave mirror version, the bulb could be seen in the foreground and may have served as a visual cue to help locate the approximate position of the aerial image. Finally, most of the poststudents had probably seen a demonstration in lecture in which an aerial image is formed by a concave mirror. The image is large, inverted, and very clearly suspended in space. As the position of the object along the mirror axis is varied, the image can be seen to be either in front of, at, or behind the object.

V. CONCLUSIONS

Many of the prestudents seemed to share one or more related ideas about the formation of real images. We will refer to these ideas as naive conceptions and summarize them in the following sequence: A well-defined, luminous object produces parallel rays of light that travel through space. As the "potential image" passes through an optical system, the various components either change it in some way, perhaps by altering its orientation or size, or provide a surface on which it can be viewed. The purpose of the lens is to invert and change the size of the image. The function of the screen is to reflect or to "capture" the light rays so that the image may be seen. An image cannot be seen in empty space, independent of a surface. All of these ideas were often either expressly stated by the prestudents or strongly implied by their diagrams on the first four converging lens tasks. The summary above may be useful as an explanatory or predictive model for student performance on similar tasks. We do not believe, however, that there is sufficient evidence to claim that these naive conceptions constitute a model for the formation of a real image that is evoked consistently by individuals untutored in geometrical optics.

Examination of the overall performance of the poststudents on the converging lens and concave mirror tasks reveals that none was successful on all of the tasks. About half of the poststudents were unsuccessful on at least half of the tasks. In attempting to perform the tasks, many of the poststudents did not spontaneously bring to bear the concepts, principles, and ray-tracing techniques that they had just studied in class. Instead, their initial responses often reflected several of the same naive conceptions found among the prestudents. Varying degrees of prompting by the investigator were sometimes necessary before the poststudents would try to apply the formalism that they had been taught to the real optical system in front of them.

It was clear from the interviews with the poststudents that it is probably not uncommon to emerge from an introductory physics course without understanding the essential role of a converging lens or a concave mirror in the formation of a real image: to bend light traveling outward in many directions from each point on the object so that it converges, as closely as possible, to a single image point. Furthermore, when light emanating from a point on an object is transmitted or reflected by a lens or mirror so that it passes through a common point, a real image exists at that point.

The failure to recognize the crucial function of the lens or mirror in forming an image was a contributing factor in most of the errors made on the interview tasks. This function was certainly not understood by the students who claimed that there would be an image on the screen without the lens or mirror. Exploratory questioning had convinced us that virtually all the poststudents could locate an image by using the lens equation and by drawing a ray diagram. However, when confronted with an actual laboratory situation, the poststudents often did not seem to be consciously aware that, for a given distance of an object from a lens or mirror, the position of the image is fixed. The students who were puzzled by the existence of an aerial image did not realize that the presence of a screen to reflect or transmit an image, or an eye in the proper position to see it, are both irrelevant to its formation.

A lack of understanding of the role of the eye further exacerbated the problems the students had with the idea of observing a real image without a screen. Their lack of experience in directly viewing such an image, coupled with the frequency with which they had seen one on a screen, may have predisposed some of the students always to associate the two together. Those who may have recognized that the screen was not necessary often could not decide where the eye of the observer should be placed. The students' experience in drawing ray diagrams did not seem to provide much help in this case. Usually in such diagrams rays are not drawn beyond the image point, and even when they are little significance may be attached to the extended rays. The students did not realize that to see an image the eye must be placed beyond the point of its formation so that light diverging from it can enter the eye.

The light ray was not a fully developed concept for many of the students. In attempting to justify their predictions, they often referred to a light ray as if it were a physical entity that actually forms an image rather than a geometrical representation that is useful for describing how light behaves under certain circumstances. Instead of treating the ray diagram as a geometrical algorithm, the students frequently seemed to endow it with a separate existence. For example, many students apparently did not understand the significance of the three special rays that are used in a ray diagram to locate the image. Often their comments indicated a belief that these rays are necessary for forming an image. Consequently they might claim that blocking off the top two of the special rays would lead to a disappearance of part of the image. These students did not realize that any two of these rays are sufficient to locate the position of the image and that any other two rays would serve just as well. The special rays are convenient because of the simple rules for tracing their paths.

When the poststudents attempted to justify their responses on the tasks by drawing a ray diagram, they often failed to recognize its universality for a given separation between the object and the lens or mirror. Although basically there was only one demonstration for the converging lens tasks and only one for the concave mirror tasks, many of the students seemed to think they needed different ray diagrams for the various tasks. They were often unable to represent the proposed changes in the components of the optical system on the diagram, make an appropriate analysis, and interpret the results to make a correct prediction. For example, in deciding what would happen to the image if the screen position were to change, many of the students maintained that the image would remain on the screen. They did not seem to realize that the screen does not appear explicitly on a ray diagram. The position of the object with respect to the lens or mirror is all that is involved. When light rays from a point on an object strike a lens or mirror and then intersect at a common point, a real image exists. Only when the screen is located at the same, or nearly the same, place as the real image will an observer be able to see anything but a lighted screen.

Even when the students succeeded in producing an appropriate ray diagram, they were often at a loss about how to extract the information necessary for performing a particular task. Moreover, students who could find the position of an image by ray-tracing frequently seemed unaware of the limitations of this procedure. To respond to certain questions, it is necesssary to be able to invoke information not explicitly contained in a ray diagram. For example, such a diagram does not usually specify where an observer's eye must be placed in order to see an image; nor can the brightness or clarity of an image be inferred from a straightforward application of ray-tracing procedures.

Implications for instruction

There is often a tacit assumption that students who have performed satisfactorily in the geometrical optics portion of an introductory physics course can respond correctly to the basic questions presented at the beginning of this paper. The discussion above demonstrates that, although they might have been able to give correct verbal responses to these questions, the students who participated in our study were frequently unable to relate their knowledge to simple, but real, optical systems.

During the individual demonstration interviews, we identified a number of specific conceptual difficulties exhibited by the students as they attempted to perform a series of tasks designed to probe their understanding of image formation. Our experience during the interviews suggests that one way to begin addressing these difficulties is to incorporate into the instructional process tasks similar to those administered in the investigation. The following two examples are offered as illustrations of how the tasks may be included in a course as lecture demonstrations or laboratory activities. To help students understand that the special rays are not necessary for forming an image but merely sufficient to locate its position, the instructor might perform a demonstration in which various portions of a lens or mirror are covered with a mask while students examine the effect on the image. Directions for an open-ended laboratory investigation involving lenses and mirrors could include a suggestion to explore the relationship between the location of an image and the placement of a screen. The experience of moving a screen back and forth along the principal axis of an optical system can develop conscious awareness that no image will be seen unless the screen is placed at a particular position. Similarly, looking along the axis, while moving the screen in and out of the system in a direction transverse to the axis, can help convince students of the existence of an aerial image.[15]

The instructional strategies suggested above share the common characteristic of encouraging students to think about issues that they might not consider without active intervention on the part of the instructor. If from either research or teaching, we know that a particular aspect of some topic is likely to be a source of confusion, it is important that students become actively engaged in confronting and resolving the matter. Otherwise a misconception may remain undetected and, if sufficiently basic, may preclude further meaningful learning. Certainly some of the difficuties with geometrical optics that we identified among the students in our study fall into this category.

Underlying our investigation of how students think about image formation is the point of view that for scientific concepts to be useful to students, they should be able to apply these concepts to actual objects and events. We have found that this level of understanding of geometrical optics often does not develop spontaneously through traditional teaching in which the emphasis is on solving numerical problems. There seems to be a need for greater emphasis on developing a qualitative understanding of ideas that on the surface may appear too trivial to warrant special attention.

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