Ph 161 Black Holes

Homework Assignment 1

Due Tuesday, January 23, 2006

This should be your own work; do not copy problem solutions.

(1.) Frame $\bar{\mathcal{O}}$ is boosted along frame $\mathcal{O}$'s x-axis by speed v. By consulting the spacetime diagrams in the lecture slides, give a geometric proof that $\bar{\mathcal{O}}$'s $\bar{x}$-axis (the locus of events *simultaneous* with event $\bar{t} = 0$, $\bar{x} = 0$ in frame $\bar{\mathcal{O}}$) as plotted in frame $\mathcal{O}$ corresponds to the equation $t = v\,x$ (where we have set $c = 1$, of course).

(2.) Consider one observer (frame $\mathcal{O}$) who measures the length of a rod at rest, laying on the x-axis in this frame to be $l = 5\,\mathrm{m}$. Another observer (frame $\bar{\mathcal{O}}$) moves with constant speed $v = 0.9$ (that is, 90% the speed of light) along $\mathcal{O}$'s x-axis. What does he measure for the length of this rod? Illustrate with a spacetime diagram why these observers measure different lengths.

(3.) Describe in words and explain how the Einstein summation convention works. We gave an example of a matrix representation for a simple Lorentz boost along one of the spacelike axes. One should be wary of these matrix representations because they can get you in trouble if you do not pay close attention to how the sums work in the Einstein summation convention. For example, can you come up with a matrix representation for the double sum $ds^2 = \eta_{\alpha\beta}\, dx^\alpha\, dx^\beta$?

(4.) Hartle Chapter 4: problem 13.

(5.) Hartle Chapter 4: problem 15.

(6.) Hartle Chapter 5: problem 1.

(7.) Hartle Chapter 5: problem 4.

(8.) Hartle Chapter 5: problem 7.

(9.) Hartle Chapter 5: problem 20.

(10.) A photon has no four-velocity, of course, but it does have a four-momentum $\mathbf{p}$: after all, it carries energy and three-monetum. Give an argument for why $\mathbf{p}$ must be tangent to the photon's world line. What does this imply for the photon's rest mass?