

$$H = p\dot{q} - L \quad \text{Hamiltonian}$$

$$H = \frac{p^2}{m} - \frac{p^2}{2m} + V(q) = \frac{p^2}{2m} + V(q) \quad \text{energy}$$

quantum theory $[q, p] = i\hbar$

oscillator example $V(q) = \frac{1}{2} m\omega^2 q^2$ operators

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

$$a^+ = \sqrt{\frac{m\omega}{2\hbar}} \left(q - i \frac{1}{m\omega} p \right)$$

creation and annihilation operators

$$a = \sqrt{\frac{m\omega}{2\hbar}} \left(q + i \frac{1}{m\omega} p \right)$$

$$[a, a^+] = 1$$

$$H = \hbar\omega \left(a^+ a + \frac{1}{2} \right)$$

↑ zero point energy (can be left out by normalization)

$$a |0\rangle = 0 \quad \text{ground state}$$

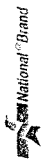
note on gravity and cosmological constant

$$a^+ |0\rangle \quad \text{first excited state}$$

$$|n\rangle = \frac{(a^+)^n}{\sqrt{n!}} |0\rangle \quad H |n\rangle = \left(n + \frac{1}{2} \right) \hbar\omega |n\rangle$$

$$\langle \psi(t) | a | \phi(t) \rangle = \langle \psi(0) | e^{\frac{i}{\hbar} Ht} a e^{-\frac{i}{\hbar} Ht} | \phi(0) \rangle$$

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$$a(t) = e^{\frac{i}{\hbar} H t} a e^{-\frac{i}{\hbar} H t} \quad a(0) = a$$

Heisenberg picture

$$\frac{da(t)}{dt} = \frac{i}{\hbar} [H, a]$$

$$\dot{a}(t) = -i\omega a(t)$$

$$a(t) = e^{-i\omega t} a$$

$$a^+(t) = e^{i\omega t} a^+$$

$$q(t) = \sqrt{\frac{\hbar}{2m\omega}} \left(a e^{-i\omega t} + a^+ e^{i\omega t} \right)$$

satisfies the Euler-Lagrange equation

$$m\ddot{q} = -m\omega^2 q$$

classically a, a^+ are determined
by initial conditions $q(0)$ and $p(0)$

$$\ddot{q} + \omega^2 q = 0$$

generalization to N degrees of freedom

$$S = \int_{t_i}^{t_f} L(q_k, \dot{q}_k) dt$$

$$L(q_1, \dot{q}_1, q_2, \dot{q}_2, \dots, q_N, \dot{q}_N)$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_k} = \frac{\partial L}{\partial q_k} \quad k=1, 2, \dots, N$$

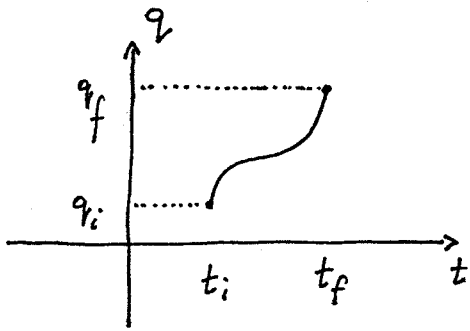
$$p_k = \frac{\partial L}{\partial \dot{q}_k} \quad k=1, 2, \dots, N \quad \text{canonical momenta}$$

$$H = \sum_k p_k \dot{q}_k - L \quad \text{Hamiltonian}$$

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Action Principle and Quantum Theory (digression)



$$S = \int_{t_i}^{t_f} L(q, \dot{q}) dt$$

↑
K-V

$\delta S = 0$ action principle in classical physics selects physical trajectory

$$\delta S = \int_{t_i}^{t_f} [L(q + \delta q, \dot{q} + \delta \dot{q}) - L(q, \dot{q})] dt$$

$$\delta S = \int_{t_i}^{t_f} \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right) dt = \int_{t_i}^{t_f} \left(\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} \right) \delta q dt$$

by partial integration

↑
must vanish for arbitrary δq
Newton equation

in Quantum Mechanics:

$$G(q_f, q_i; t_f, t_i) = \langle q_f | e^{-\frac{i}{\hbar} H(t_f - t_i)} | q_i \rangle$$

fundamental object
(propagator)

at $t = t_i$ particle is prepared at $q = q_i$
probability amplitude (complex) that particle will
be found at $q = q_f$ at some later time $t = t_f$

$$\langle \psi_f | e^{-\frac{i}{\hbar} H t} | \psi_i \rangle = \int dq' \int dq \psi_f^*(q') G(q', q; t_f - t_i) \psi_i(q)$$

at $t = t_i$ $|\psi_i\rangle$ general initial state

$t = t_f$ $|\psi_f\rangle$ probability amplitude of some final state

$$G(q_f, q_i; t_f - t_i) = \sum_{\text{all paths}} e^{\frac{i}{\hbar} S(\text{path})}$$

Quantum Action Principle
Feynman path integral
(contains classical limit)

$\frac{S}{\hbar}$ phase of a path

$$\frac{\hbar}{h} = \frac{h}{2\pi} = 1.0546 \times 10^{-34} \text{ Js} = 6.5822 \times 10^{-22} \text{ MeVs}$$

Planck's constant

microscopic motion $S \lesssim \hbar$ all paths contribute
(quantum regime)

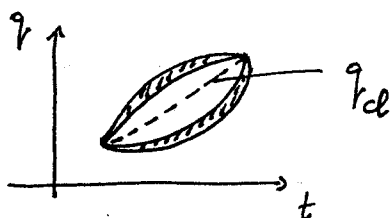
electron moving over distance $q_f - q_i = 0.1 \times 10^{-8} \text{ cm}$ (size of atom)
or smaller

$v = 0.1c$ nonrelativistic

$$t_f - t_i = \frac{0.1 \times 10^{-8} \text{ cm}}{0.3 \times 10^{10} \frac{\text{cm}}{\text{s}}} \approx 3 \times 10^{-18} \text{ s}$$

$$S = \frac{1}{2} \underbrace{0.5 \text{ MeV}/c^2}_{m_e} \cdot (0.01 c^2) \cdot 0.3 \times 10^{-18} \text{ s} = 7 \times 10^{-22} \text{ MeVs}$$

$\frac{\delta S}{\hbar}$ phase is slowly changing in quantum motion over macroscopically measurable spread of path bundle, nonclassical trajectories all contribute in quantum situation



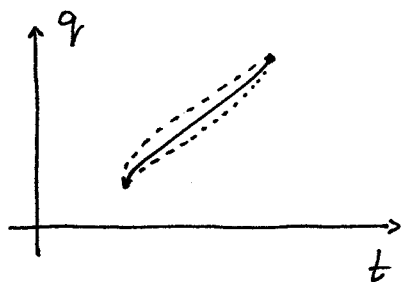
macroscopic body (classical limit)

$$m = 1 \text{ gr} \quad (m_e = 9.1 \times 10^{-28} \text{ gr})$$

$$v = \frac{1}{10} c$$

$$l = 1 \text{ cm}$$

$$S \sim 10^{17} \text{ MeV} \cdot \text{s} \quad \sim 10^{40} \hbar$$



only very narrow band around the classical path contributes