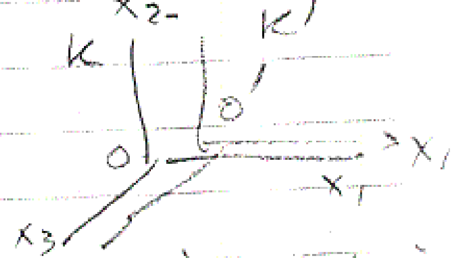
14. Special Theory of Relativity

1. Galilean Invariance

Despite misconceptions to the contrary, a principle of relativity was articulated by Galileo centuries before Einstein. He said that phenomena obey the same laws of motion in any uniformly moving reference frame (inertial frame). One says that the EOM are invariant to uniform translation.

Consider 2 coordinate systems, the unprimed at rest, the primed moving with speed v along the x_1 axis. Arrange things so that O and O' coincide at $t = 0$.



Then

$$x_1' = x_1 - vt$$
$$x_2' = x_2$$
$$x_3' = x_3$$

and, if we believe in universal time

$$t' = t. \quad \text{(wrong?)}$$

This is called a Galilean transformation.

⟹ Lengths are clearly unchanged since

$$x_1'(a) - x_1'(b) = x_1(a) - x_1(b)$$

$$\therefore \sum_j dx_j^2 = \sum_j dx_j'^2$$

Newton's laws are invariant to Galilean transformation, since

$$\dot{x}_1' = \dot{x}_1 - v$$

$$\ddot{x}_1' = \ddot{x}_1$$

If we assume (incorrectly)

$$m' = m$$

then

$$F_j' = m'\ddot{x}_j = m\ddot{x}_j = F_j.$$

2. Constancy of speed of light

Newton's view of the world, including assumptions about space and time began to unravel when it was shown convincingly by Michelson & Morley (1881–1887) that the S.O.L. was constant independent any relative motion of source and observer. This is clearly inconsistent with Galilean law for velocity transform. If a light beam is moving with speed c in K frame, then in K' frame, it moves with speed

$$c' = c - v$$

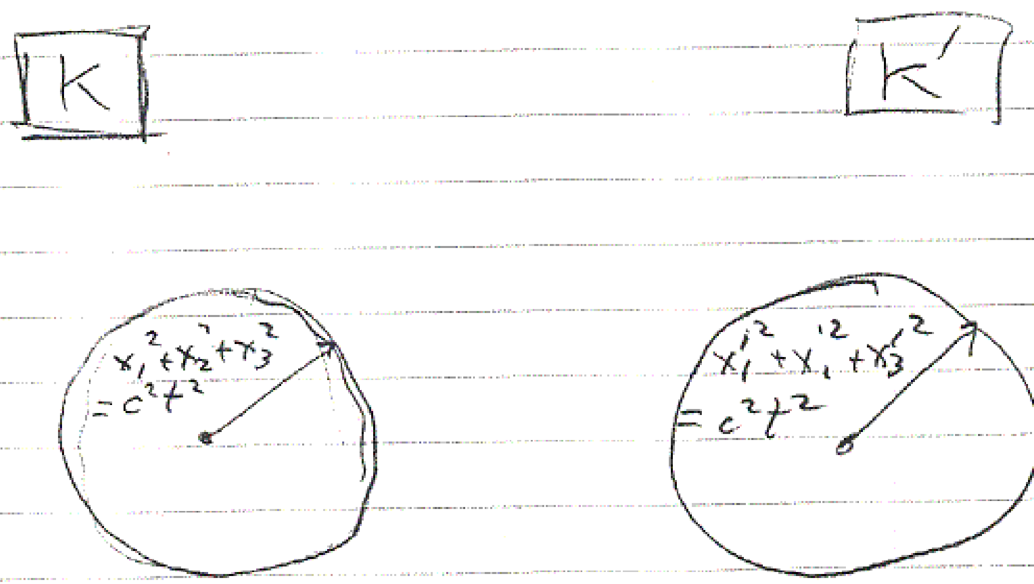
But this is not found, ever!

what is found, is that, as seen in both frames, a flash of light will propagate in all directions at the SoL. If

If $\begin{cases} x_j(t) \text{ is pos. of light front, at time } t \text{ in } K \\ x_j'(t') \text{ " " " " " } t' \text{ in } K' \end{cases}$

then it is found experimentally

$$\sum_{j=1}^{3} x_j^2 - c^2 t^2 = 0$$
$$\sum_{j=1}^{3} x_j'^2 - c^2 t'^2 = 0 \qquad (14.5)$$



How is this possible?

⇒ Have to abandon notion of universal time.

⇒ time is local and relative

3. Einstein's Postulates of SR

1) physical laws are the same in all IRF (i.e., mathematical form is same)
→ Concept of absolute rest is meaningless
→ only relative motions can be measured

2) the speed of light is a universal constant independent of any relative motion of source and observer

1) is a re-affirmation of the Galilean notion of relativity
2) is a statement of observational fact.

These two, apparently contradictory postulates can be reconciled if we give up current notions about space and time.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

If we allow distances and time to change in the same way in a moving reference frame, then the SOL can remain unchanged.

4. Death of universal time/simultaneity

What do we really mean by time?

It is the reading on a clock when 2 events occur simultaneously. For example If a clock reads 10:00 when the train pulls into the station, we say the train arrives at 10:00.

To see that time is local, imagine synchronizing the clock and train, so that the clock reads exactly 10:00 when the train arrives (this is a German train). Then, transport the clock to the moon. What will the clock read when the train pulls into the station? ~~It~~ depends. on where you make ~~the~~ observation.

⇒ station: clock reads 10:00 - 1 1/4 sec

⇒ moon ? clock reads 10:00 + 1 1/4 sec

Only if the clock is <u>in the station</u> will it read exactly 10:00 when the train arrives.

The arrival of the train and the stat clock turing 10:00 constitute an <u>event</u> in spacetime with coordinates

$$\{x_j, t\}$$

The train's arrival and the moon clock turing 10:00 constitute 2 different events in spacetime $\{x_j, t\}_{sta}$ and $\{x'_j, t'\}_{moon}$

They are related by 14.5.

4. Lorentz transformation

Lorentz showed that a special trans. between (x, t) and (x', t') left Maxwell's equations invariant. The LT is the correct transformation for relativistic dynamics, supplanting the Galilean transformation. The LT governs how observers moving relative to one another measure positions and times.

To derive, imagine flash bulb goes off when K and K' coincide at the origin. By construction

$$\left.\begin{aligned} x_2' &= x_2 \\ x_3' &= x_3 \end{aligned}\right\}$$

Let $t = t' = 0$ when bulb goes off, and let K' move to right w/ speed v

O' wrt K: $x_1 - vt = 0$

O' wrt K' $x_1' = 0$

Since GT is incorrect, assume

$$x_1' = \gamma(x_1 - vt) \qquad (14.9)$$

γ must $\to 1$ as $v/c \to 0$

Relative to O', R is moving to left with speed v

$$x_1 = \gamma'(x_1' + vt') \qquad (14.10)$$

Postulate I demands $\gamma = \gamma'$.
Substituting for x_1' from above

$$x_1 = \gamma[\gamma(x_1 - vt) + vt']$$
$$= \gamma^2 x_1 - \gamma^2 vt + \gamma vt'$$

$$(1-\gamma^2)x_1 + \gamma^2 vt = \gamma vt'$$

$$\boxed{t' = \gamma t + \frac{(1-\gamma^2)}{v\gamma} x_1} \qquad (14.11)$$

Postulate II demands

$$\left.\begin{aligned} x_1 &= ct \\ x_1' &= ct' \end{aligned}\right\} \qquad (14.12)$$

After some algebra,* we find

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \qquad (14.13)$$

The complete LT can be written

$$\boxed{\begin{aligned} x_1' &= \gamma(x_1 - vt) & x_2' &= x_2 \\ t' &= \gamma\left(t - \frac{vx_1}{c^2}\right) & x_3' &= x_3 \end{aligned}}$$

The inverse transformation is easily obtained by letting $v \to -v$ and exchanging primed and unprimed

$$\boxed{\begin{aligned} x_1 &= \gamma(x_1' + vt') & x_2 &= x_2' \\ t &= \gamma\left(t' + \frac{v x_1'}{c^2}\right) & x_3 &= x_3' \end{aligned}}$$

These transformations govern how events in spacetime are related.

$$K: \{x_j, t\} \overset{LT}{\Longleftrightarrow} K': \{x_j', t'\}$$

The key to understanding paradoxes is to think of events, rather than positions and times separately.