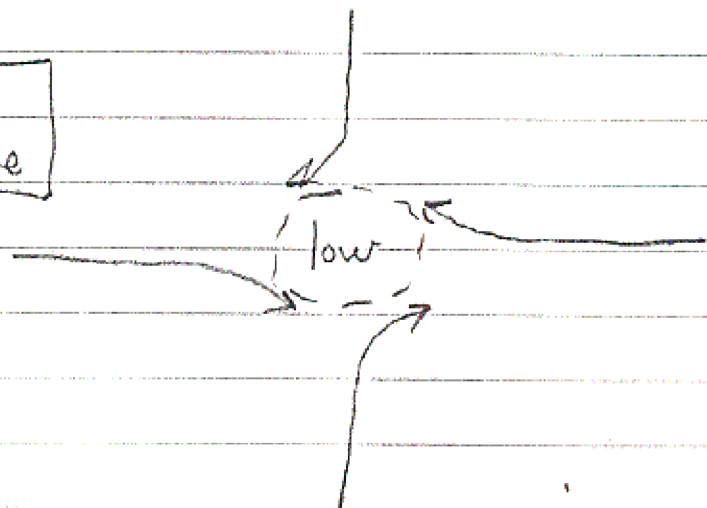
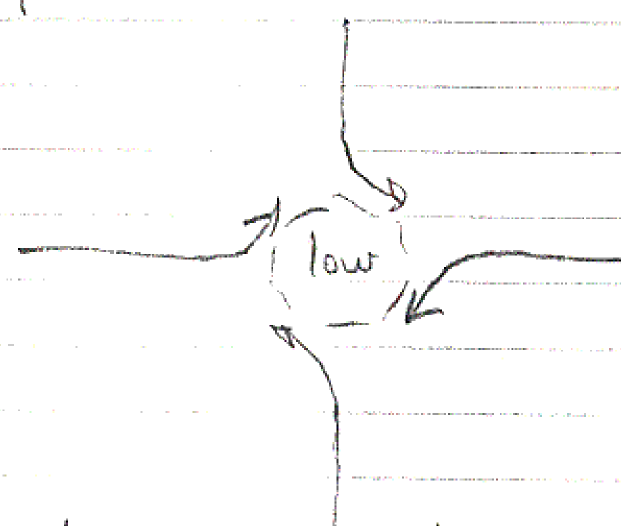
drive a flow from high to low pressure. Flow will be deflected to the right, creating counter-clockwise cyclonic motion in northern hemisphere

Northern hemisphere



In the southern hemisphere deflection is to the left, because $\omega_z < 0$. This results in clockwise cyclones in the south

Southern hemisphere

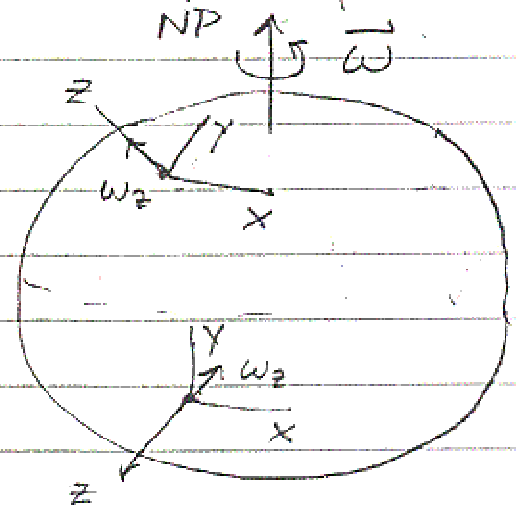


At the equator, $\vec{\omega} \cdot \hat{e}_z = 0$, and Coriolis force vanishes.

Coriolis' force effects

The Earth rotates from W to E: counterclockwise when seen from the North pole. $\vec{\omega}$ is therefore pointing N

At some northern latitude, component of $\vec{\omega}$ normal to surface is $+$; in the south $-$



Define rotating coord. sys. with $\hat{e}_z$ along local outward normal to surface.

In northern hemisphere

$$\vec{\omega} \cdot \hat{e}_z \equiv \omega_z > 0$$

assume $\vec{v}_r$ is along x axis

$$-m\, \vec{\omega}_z \times |v_r| \hat{e}_x \text{ is in } -\hat{e}_y \text{ direction}$$

assume $\vec{v}_r$ is along y axis

$$-m\, \vec{\omega}_z \times |v_r| \hat{e}_y \text{ is in } \hat{e}_x \text{ direction}$$

$\Rightarrow$ particle is deflected to the right in every case. Consider a low pressure region in the Earth's atmosphere. Pressure gradient will

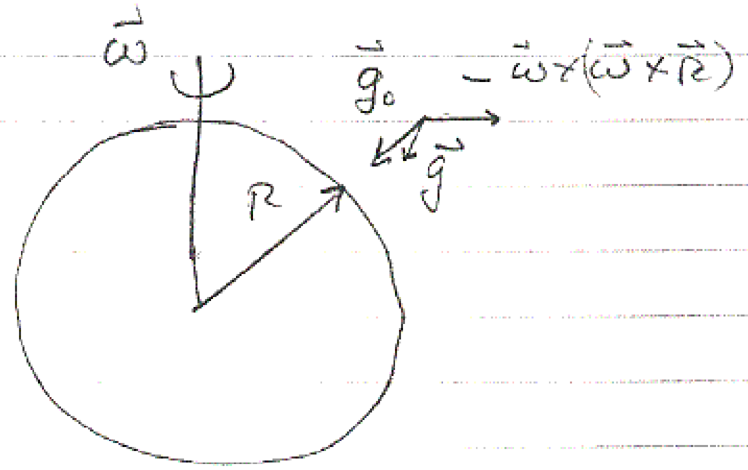
Centrifugal force term is dominated by $\vec{\omega}\times(\vec{\omega}\times\vec{R})$ term.
Let's calculate magnitude of term:

$$|\vec{\omega}\times(\vec{\omega}\times\vec{R})| \sim \omega^2 R$$

$$\omega = 7.3\times10^{-5}\ \text{rad/s}$$

$$\omega^2 R = (7.3\times10^{-5})^2(6.4\times10^{6}\ \text{m})$$
$$= 3.4\times10^{-2}\ \text{m/s}^2$$

This is 0.35% of $|\vec{g}_0|$, and non-negligible.
The direction of $\vec{\omega}\times(\vec{\omega}\times\vec{R})$ is away from rotation axis.



$\vec{g} \parallel \vec{g}_0$ at pole and equator.

@ pole, $\vec{g} = \vec{g}_0$

@ equator $\vec{g} = \vec{g}_0\left(1 - \dfrac{\omega^2 R^3}{GM_E}\right)$

$\Rightarrow$ gravity is "weaker" at equator by 0.35%.

$\vec{R}$ is coord. of O in IRF, and is time-dependent.
$\vec{r}$ is coord. of point P in RRF
$\vec{v}_r$ is velocity " " P in RRF

Using $\left.\frac{d\vec{Q}}{dt}\right)_f = \left.\frac{d\vec{Q}}{dt}\right)_r + \vec{\omega}\times\vec{Q}$

we have

$$\left.\frac{d\vec{R}}{dt}\right)_f = \cancelto{0}{\left.\frac{d\vec{R}}{dt}\right)_r} + \vec{\omega}\times\vec{R}$$

$$\left.\frac{d^2\vec{R}}{dt^2}\right)_f = \ddot{\vec{R}}_f = \vec{\omega}\times\left.\frac{d\vec{R}}{dt}\right)_f$$

$$= \vec{\omega}\times(\vec{\omega}\times\vec{R})$$

and so substituting and combining terms

$$\vec{F}_{eff} = \vec{S} + m\vec{g}_0 - m\vec{\omega}\times[\vec{\omega}\times(\vec{r}+\vec{R})] - 2m\vec{\omega}\times\vec{v}_r$$

Define effective gravity $\vec{g}$

$$\vec{g} \equiv \vec{g}_0 - \vec{\omega}\times[\vec{\omega}\times(\vec{r}+\vec{R})]$$

then

$$\boxed{\vec{F}_{eff} = \vec{S} + m\vec{g} - 2m\vec{\omega}\times\vec{v}_r}$$

Note $\vec{g}$ varies from place to place on the surface of the Earth. For motions near the surface of the Earth ($|\vec{r}| \ll |\vec{R}|$)

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4. Motion relative to the Earth

Motion on the surface of the Earth is dominated by its rotation and its gravity. To a very good approximation, $\dot{\vec{\omega}} = 0$, hence

$$\vec{F}_{eff} \equiv m\vec{a}_r = \vec{S} + m\vec{g}_0 - m\ddot{\vec{R}}_f - m\vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2m\vec{\omega}\times\vec{v}_r$$

where

$$\vec{g}_0 = -G\frac{M_E}{R^2}\hat{e}_R$$

and $\vec{S}$ is the sum of all other body forces (e.g., pressure, EM, etc.).

To evaluate 3rd term, establish a rotating coord. syst. XYZ at some point fixed to the surface of the Earth