current in a semiconductor diode is approximately proportional to the forward bias voltage (square-law detection), for relatively small values of forward bias voltage, and since this current appears in the load resistor, then the voltage across the load resistor is approximately proportional to the square of the voltage appearing at the input to the diode detector.

[6]For a relatively detailed discussion of the field distribution associated with a continuous aperture (applicable to a circular dish) including some discussion of the Reciprocity Theorem (enabling the dish to be considered either as a transmitting or receiving antenna), see Kraus, *Radio Astronomy, op.cit.* p. 165 ff; John D. Kraus and Keith R. Carver, *Electromagnetics* (McGraw-Hill, New York, 1973), 2nd ed., p. 668 ff. For a more explicit treatment of the circular dish, including an expression for the beamwidth, see Samuel Silver, Ed. *Microwave Antenna Theory and Design* (Boston Technical, Boston, MA, 1965), p. 192 ff.

[7]It should be noted that all three terms in Eq. (2) represent noise in the sense that each term represents a random temporal distribution. Hence, the terms are uncorrelated and are taken to combine as indicated. The noise power from the Sun is being called "signal" power on the basis that it is the significant quantity in the experiment.

[8]The beamwidth of the antenna could, in principle, be calculated, but would require a detailed knowledge of the antenna system, especially of the illumination of the dish by the feed horn. For further discussion of this point, see Kraus, *Radio Astronomy, op.cit.* p. 212 ff.

[9]See Kraus, *Radio Astronomy, op.cit.* p. 237. Note: this section of the book is by Martti E. Tiuri.

Zero angular momentum turns

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Simple explanations are given of three examples of zero angular momentum turns. Humans and cats free of external torques can use internal forces to move parts of their bodies to produce angular momenta $\mathbf{L}_1$ and $\mathbf{L}_2$ of the parts, whose vector sum may not be zero. Conservation of angular momentum then requires an $\mathbf{L}_3$ of the whole body so that $\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 = 0$. This can produce a rotation or partial rotation of the body in the absence of any external torque or initial angular momentum.

I. INTRODUCTION

There seems a curious lack of clear and simple explanations of zero angular momentum turns, or orientational changes, which can readily be produced, for example, by falling cats, or astronauts, or divers or trampolinists. Some physicists seem to insist, quite incorrectly, that such midair turns are not even possible! Kenneth Laws,[1] in a recent article in Physics Today on "The Physics of Dance" even says

"For example, no matter how much a dancer may wish to leap off the floor and *then* start turning (say for a *tour jeté*—that is, a turning leap), the law of conservation of angular momentum absolutely prevents such a movement."

Jearl Walker[2] also says

"In jeté en tournant, a turning leap, the dancer jumps into the air with no apparent spin about her vertical axis, yet near the top of the leap she begins to rotate. Impossible. One of the firm laws of physics is that the angular momentum of an object remains constant unless a torque acts on the object. If the dancer was not spinning when she left the floor, she cannot begin to spin in midair."

Walker claims

"The explanation of the illusion is that the dancer does have a small spin at the beginning of the leap ... too slight for an observer to notice it."

It seems as though Laws and Walker only believe angular momentum can be conserved when it is not zero!

Of course the photographic evidence proving that cats, gymnasts, divers, and astronauts, can change their orientation in midair even when they have no initial angular momentum, and no external torques, is readily available. The detailed explanations of such rotations or partial rotations merely require a consideration of the conservation of vector angular momentum at a value of zero, so that both $\mathbf{L} = 0$ and $d\,\mathbf{L}/dt = 0$.

Here are three such cases, where internal forces can produce orientational changes, or turns, while no external torques are applied.

II. HUMAN BODY ROTATION FORWARD OR BACKWARD

If a nonrotating person, free of external forces and torques with arms extended straight out to the sides, starts moving his arms in forward circles as if swimming the butterfly stroke, the arms acquire a vector angular momentum to his left. To conserve vector angular momentum at its original value of zero, the entire body must acquire an equal and opposite angular momentum to his right. This requires a head-backward feet-forward body rotation. This rotation of course only lasts as long as the arms are "paddled." If the sense of rotation of the arms is reversed the sense of rotation of the body will also reverse.[3] (So if a diver had enough time in the air, he could do backward somersaults followed by forward somersaults.)

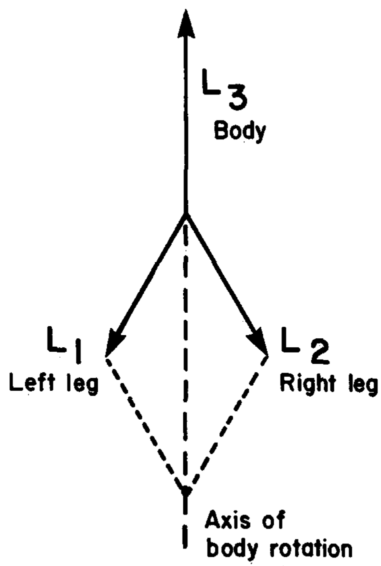

Fig. 1. Angular momentum of a dancer moving her legs in clockwise circles (instantaneous back view).

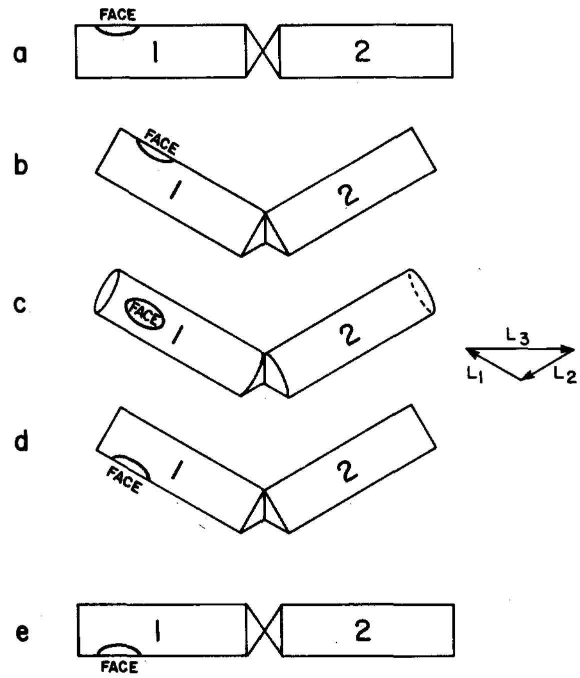

Fig. 2. Stylized falling cat, shown as two cylinders with conical ends in contact. (a) Spine straight, before release. (b) Spine bowed, upon release. (c) Rotation in progress, showing angular momentum diagram. (d) Spine arched, turn completed. (e) Spine straight for landing.

III. HUMAN BODY ROTATION LEFT OR RIGHT

If an astronaut, or a dancer in midair, free of external torques, extends her right leg straight forward and left leg straight back, and then moves each foot in a clockwise (as seen from above) semicircle, the legs will acquire angular momenta whose vector sum will be pointing down her body. To conserve zero angular momentum, her whole body must turn counterclockwise, with an angular momentum equal and opposite to the angular momentum of the two legs. The body turn will stop as soon as the twisting "scissors kick" is completed.[3]

If clockwise circles, not semicircles, are described on each side by each foot, the instantaneous angular momentum vector diagram of the dancer (seen from directly behind), will look like Fig. 1. The angular momenta of the left and right legs are represented by $\mathbf{L}_1$ and $\mathbf{L}_2$. The body must therefore acquire an angular momentum $\mathbf{L}_3$ so that $\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 = 0$. The axis of the dancer's body rotation will be along $\mathbf{L}_3$. Note that while both $\mathbf{L}_1$ and $\mathbf{L}_2$ rotate around the body-rotation axis, their vector sum determines the body-rotation axis, since in the absence of external torques $d\,\mathbf{L}/dt$ must remain zero. The body will continue to rotate as long as the legs are kept moving in circles.

Clearly a dancer may leap up with no initial angular momentum, and *then* turn in midair by some variation of the twisting scissors kick technique.

IV. A FALLING CAT

It is also well known that a cat, dropped back-first, can quickly turn over and land on its feet.[4] This rapid change of orientation is another example of the conservation of vector angular momentum which remains zero as the cat turns. The ability to flex and pivot with a "hula hoop" motion about the "waist" is required. Consider the body as two cylinders with conical ends in contact[5,6] at the waist. To start turning one must make the two parts roll along their line of contact. Each cylinder has angular momentum when it is rolling about its own axis. This alone would produce angular momenta $\mathbf{L}_1$ and $\mathbf{L}_2$ whose vector sum would be to the left, say. Then there must be a counter-rotation of the whole body to produce an $\mathbf{L}_3$ to the right so that $\mathbf{L}_1 + \mathbf{L}_2 + \mathbf{L}_3 = 0$. Figure 2 is a stylized sketch of a falling cat. Upon release the cat bends at the waist so that its back is concave down. Then the cat must roll parts 1 and 2 of its body along their line of contact. If the contact line between the cones is "joining" away from us, angular momenta $\mathbf{L}_1$ and $\mathbf{L}_2$ will be as shown. This requires an angular momentum $\mathbf{L}_3$ for the whole cat to the right as shown, and the cat's face will be turning toward us. Note that by the time its face is down, the back will be arched so that the cat's belly is concave down. When one-half rotation has occurred, the cat straightens its back and stops turning, in time to land on its feet. (Note that the essential features of the explanation do *not* require any consideration of the leg bending or stretching which produce changes in the rotational inertias of the two halves of the cat during the turn. Note also that tail movement alone, in a propellerlike rotation, could cause a slow counterrotation of the cat's body.)

[1] Kenneth Laws, Phys. Today **38**(2), 24 (1985).

[2] Jearl Walker, Sci. Am. **246**(6), 146 (1982).

[3] This action is well illustrated by astronauts in the Skylab AAPT Films "Human Momenta," and "Moving Astronauts."

[4] This action is shown clearly, for example, in Alvin Hudson and Rex Nelson, *University Physics* (Harcourt Brace Jovanovich, New York, 1982), p. 246.

[5] Cliff Frohlich, Am. J. Phys. **47**, 583 (1979).

[6] Cliff Frohlich, Sci. Am. **242**(3), 154 (1980).